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Wave Propagation in a Generalized Thermoelastic Transversely Isotropic Plate Using Eigenvalue Approach

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In the present work, frequency equations are obtained for Rayleigh-Lamb wave propagation in a transversely isotropic plate of thermoelastic material, in the context generalized theory of thermoelasticity. The thickness of the plate is taken to be finite and the faces of the plate are assumed isolated and free from stresses. The analytical solution for the temperature, displacement components and stresses are obtained by using the eigenvalue approach. The frequency equation corresponding to the symmetric and antisymmetric modes of wave propagation of the plate are obtained. The function iteration numerical scheme is used to solve the complex frequency equation, in order to obtain phase velocity and attenuation coefficients of propagating wave mode. The results have been verified numerically and are represented graphically.

Keywords: Eigenvalue Approach, Wave Propagation, Transversely Isotropic Plate, Frequency Equation.

RESEARCH ARTICLE

1. INTRODUCTION

During the last three decades, non-classical theories of thermoelasticity so called generalized thermoelasticity have been developed in order to remove the paradox of physically impossible phenomenon of infinite velocity of thermal signals in the conventional coupled thermoelasticity. Lord-Shulman theory[1] and Green-Lindsay theory[2] are important generalized theories of thermoelasticity that become center of interest of recent research in this area. In Lord-Shulman theory, a flux rate term into the Fourier's law of heat conduction is incorporated (with one relaxation time) and formulated a generalized theory admitting finite speed for thermal signals. The new theory, which has been named the 'Generalized Theory of Thermoelasticity' and received much attention,[3–5] eliminates the paradox of an infinite velocity of propagation and admits finite speed for the propagation of thermoelastic disturbances. This is due to their many applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. Second, in the nuclear field, the extremely high temperature and temperature gradients originating inside nuclear reactors influence their design and operations. Moreover, it is recognized that the thermal effects on the elastic wave propagation supported by elastic foundations may have implications related to many seismological applications. Applying the above theories of generalized thermoelasticity, several problems have been solved by finite element method and analytical method.[6–13] Recently,[14–47] have considered different problems by using the generalized thermoelasticity theories. The propagation of generalized thermoelastic waves in plates of isotropic media has considered by Puri.[48] Verma and Hasebe[49] studied the propagation of thermoelastic waves in infinite plates in the context of generalized thermoelasticity and linear theory of thermoelasticity without energy dissipation. Deresiewicz[50] studied the propagation of waves in thermoelastic plates under plain strain state. Agarwal[51–53] investigated the wave propagation in generalized thermoelasticity and electromagneto-thermoelastic medium. Daimaruya and Naitoh[54] have considered the dispersion and energy dissipation of thermoelastic waves in a plate. Kumar and Partap[55] studied symmetric and skew symmetric wave mode for free vibration of microstretch thermoelastic plate with one relaxation time. Tumar[56] studied the wave propagation in a micropolar elastic plate with voids.

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In this paper, the propagation of Rayliegh-Lamb waves in a transversely isotropic plate of thickness $2d$ is studied based on Lord and Schulman theory. The analytical solution for the temperature, displacement components and stresses are obtained by using the eigenvalue approach. The frequency equations corresponding to the symmetric and antisymmetric thermoelastic modes of wave are obtained. The phase velocity and attenuation coefficients for the first five modes of waves are represented graphically. Interesting feature in this work is that the eigenvalue approach gives exact solution without any assumed restrictions on the actual physical quantities.

2. FORMULATION OF THE PROBLEM

We consider an infinite transversely isotropic thermally conducting elastic plate with uniform thickness $2d$ and temperature T_0 in the undisturbed state initially. The z-axis is the axis of the symmetry for the axisymmetric plate. We take the origin of the coordinate system (x, y, z) on the middle surface of the plate. The x–y-plane is chosen to coincide with the middle surface and z-axis normal to it along the thickness as illustrated in Figure 1.

The two-dimensional stress equations of motion and heat conduction equation in the absence of body force and heat source for a linearly elastic medium are

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = \rho \frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} \tag{2}$$

$$K_1 \frac{\partial^2 T}{\partial x^2} + K_3 \frac{\partial^2 T}{\partial z^2} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(\rho c_e T + T_0\left(\beta_1 \frac{\partial u}{\partial x} + \beta_3 \frac{\partial w}{\partial z}\right)\right) \tag{3}$$

with

$$\begin{aligned} \sigma_{xx} &= c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial z} - \beta_1 (T - T_0) \\ \sigma_{zz} &= c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z} - \beta_3 (T - T_0) \\ \sigma_{xz} &= c_{44}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned} \tag{4}$$

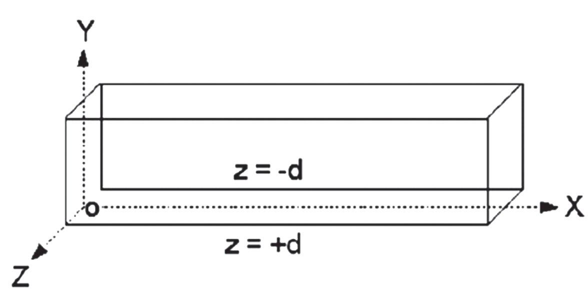

Fig. 1. Geometry of the problem.

where u and w are the displacement components along the x-axis and z-axis directions, respectively, ρ is the mass density, c_{ij} are the elastic parameters σ_{ij} are the stress components. While, T is the temperature, c_e is the specific heat capacity, K_1 and K_3 are the coefficient of thermal conductivities, τ_0 is the thermal relaxation time, t is time, $\beta_1 = (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_3$ and $\beta_3 = 2c_{11}\alpha_1 + c_{13}\alpha_3$ are the thermal stress coefficients, α_1 and α_3 are the coefficients of linear thermal expansions. For convenience, the following non-dimensional variables and notations are used:

$$\begin{aligned} (x', z', u', w') &= \frac{c}{\chi}(x, z, u, w), \quad T' = \frac{T - T_0}{T_0} \\ (t', \tau_0') &= \frac{c^2}{\chi}(t, \tau_0), \quad \sigma_{ij}' = \frac{\sigma_{ij}}{c_{11}} \end{aligned} \tag{5}$$

where $c^2 = c_{11}/\rho$ and $\chi = K_1/\rho c_e$.

Upon introducing in Eqs. (1)–(4), and after suppressing the primes, we obtain

$$\frac{\partial^2 u}{\partial x^2} + b_1 \frac{\partial^2 w}{\partial x \partial z} + b_2 \frac{\partial^2 u}{\partial z^2} - b_3 \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{6}$$

$$b_4 \frac{\partial^2 w}{\partial z^2} + b_1 \frac{\partial^2 u}{\partial x \partial z} + b_2 \frac{\partial^2 w}{\partial x^2} - b_5 \frac{\partial T}{\partial z} = \frac{\partial^2 w}{\partial t^2} \tag{7}$$

$$\frac{\partial^2 T}{\partial x^2} + b_6 \frac{\partial^2 T}{\partial z^2} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)\left(T + b_7 \frac{\partial u}{\partial x} + b_8 \frac{\partial w}{\partial z}\right) \tag{8}$$

$$\begin{aligned} \sigma_{xx} &= \frac{\partial u}{\partial x} + (b_1 - b_2)\frac{\partial w}{\partial z} - b_3 T \\ \sigma_{zz} &= (b_1 - b_2)\frac{\partial u}{\partial x} + b_4 \frac{\partial w}{\partial z} - b_5 T \\ \sigma_{xz} &= b_2\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned} \tag{9}$$

where $b_1 = (c_{13} + c_{44})/c_{11}$, $b_2 = c_{44}/c_{11}$, $b_3 = (\beta_1 T_0)/c_{11}$, $b_4 = c_{33}/c_{11}$, $b_5 = (\beta_3 T_0)/c_{11}$, $b_6 = K_2/K_1$, $b_7 = \beta_1/\rho c_e$ and $b_8 = \beta_3/(\rho c_e)$.

Now, we seek the following two-dimensional plane wave solution of the Eqs. (6)–(8)

$$(u, w, T) = (f_1(z), f_2(z), f_3(z))e^{i(qx - \omega t)} \tag{10}$$

where q is the wave number and ω is the frequency. Using Eq. (10) in Eqs. (6)–(8), we get

$$\frac{d^2 f_1}{dz^2} = b_{41} f_1 + b_{43} f_3 + b_{45} \frac{df_2}{dz} \tag{11}$$

$$\frac{d^2 f_2}{dz^2} = b_{52} f_2 + b_{54} \frac{df_1}{dz} + b_{56} \frac{df_3}{dz} \tag{12}$$

$$\frac{d^2 f_3}{dz^2} = b_{61} f_1 + b_{63} f_3 + b_{65} \frac{df_2}{dz} \tag{13}$$

where $b_{41} = (1/b_2)(q^2 - \omega^2)$, $b_{43} = (iqb_3)/b_2$, $b_{45} = -(iqb_1)/b_2$, $b_{52} = (1/b_4)(b_2 q^2 - \omega^2)$, $b_{54} = -(iqb_1)/b_4$, $b_{56} = b_5/b_4$, $b_{61} = -((iqb_7)/b_6)\omega(i + \tau_0\omega)$, $b_{63} = (1/b_6)(q^2 - \omega(i + \tau_0\omega))$ and $b_{65} = -(b_8/b_6)\omega(i + \tau_0\omega)$.

Let us now proceed to solve the coupled differential Eqs. (11)–(13) by the eigenvalue approach proposed by Refs. [20, 57–63]. Equations (11)–(13) can be written in a vector-matrix differential equation as follows

$$\frac{d\vec{V}}{dz} = B\vec{V} \tag{14}$$

where

$$\vec{V} = \begin{bmatrix} f_1 & f_2 & f_3 & \frac{df_1}{dz} & \frac{df_2}{dz} & \frac{df_3}{dz} \end{bmatrix}^T$$

and

$$B = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ b_{41} & 0 & b_{43} & 0 & b_{45} & 0 \\ 0 & b_{52} & 0 & b_{54} & 0 & b_{56} \\ b_{61} & 0 & b_{63} & 0 & b_{65} & 0 \end{bmatrix}$$

The characteristic equation of the matrix B takes the form

$$k^6 - M_1 k^4 + M_2 k^2 + M_3 = 0 \tag{15}$$

where

$$M_1 = b_{41} + b_{52} + b_{45}b_{54} + b_{63} + b_{56}b_{65}$$

$$M_2 = b_{41}b_{52} - b_{43}b_{61} - b_{45}b_{56}b_{61} + b_{41}b_{63} + b_{52}b_{63} + b_{45}b_{54}b_{63} - b_{43}b_{54}b_{65} + b_{41}b_{56}b_{65}$$

$$M_3 = b_{43}b_{52}b_{61} - b_{41}b_{52}b_{63}$$

The roots of the characteristic Eq. (15) which are also the eigenvalues of matrix B are of the form $\pm k_1, \pm k_2, \pm k_3$. The eigenvector $\vec{X} = [X_1, X_2, X_3, X_4, X_5, X_6]^T$, corresponding to eigenvalue k can be calculated as:

$$\begin{aligned} X_1 &= -b_{45}b_{56}k^2 + b_{43}(b_{52} - k^2) \\ X_2 &= -k(b_{43}b_{54} + b_{56}(-b_{41} + k^2)) \\ X_3 &= k^2(b_{52} + b_{45}b_{54} - k^2) + b_{41}(-b_{52} + k^2) \\ X_4 &= kX_1, \quad X_5 = kX_2, \quad X_6 = kX_3 \end{aligned} \tag{16}$$

From Eq. (16), we can easily calculate the eigenvector $\vec{X}_j$, corresponding to eigenvalue k_j, $j = 1, 2, 3, 4, 5, 6$. For further reference, we shall use the following notations:

$$\begin{aligned} &\vec{X}_1 = [\vec{X}]_{k=-k_1}, \quad \vec{X}_2 = [\vec{X}]_{k=-k_2}, \quad \vec{X}_3 = [\vec{X}]_{k=-k_3}, \\ &\vec{X}_4 = [\vec{X}]_{k=k_1}, \quad \vec{X}_5 = [\vec{X}]_{k=k_2}, \quad \vec{X}_6 = [\vec{X}]_{k=k_3} \end{aligned} \tag{17}$$

The solution of Eq. (14) can be written as follows:

$$\vec{V} = \sum_{j=1}^{3} A_j \vec{X}_j e^{-k_j z} + \sum_{j=1}^{3} B_j \vec{X}_{j+3} e^{k_j z} \tag{18}$$

where A_1, A_2, A_3, B_1, B_2, and B_3 are constants to be determined from the boundary condition of the problem. Thus, the field variables can be written for x, z and t as:

$$u(x,z,t) = \sum_{j=1}^{3} A_j u_j e^{-k_j z + i(qx-\omega t)} + \sum_{j=1}^{3} B_j u_j e^{k_j z + i(qx-\omega t)} \tag{19}$$

$$w(x,z,t) = \sum_{j=1}^{3} A_j w_j e^{-k_j z + i(qx-\omega t)} - \sum_{j=1}^{3} B_j w_j e^{k_j z + i(qx-\omega t)} \tag{20}$$

$$T(x,z,t) = \sum_{j=1}^{3} A_j T_j e^{-k_j z + i(qx-\omega t)} + \sum_{j=1}^{3} B_j T_j e^{k_j z + i(qx-\omega t)} \tag{21}$$

$$\begin{aligned} \sigma_{xx}(x,z,t) &= \sum_{j=1}^{3} A_j (iqu_j - (b_1 - b_2)k_j w_j - b_3 T_j) e^{-k_j z + i(qx-\omega t)} \\ &+ \sum_{j=1}^{3} B_j (iqu_j - (b_1 - b_2)k_j w_j - b_3 T_j) e^{k_j z + i(qx-\omega t)} \end{aligned} \tag{22}$$

$$\begin{aligned} \sigma_{zz}(x,z,t) &= \sum_{j=1}^{3} A_j ((b_1 - b_2)iqu_j - b_4 k_j w_j - b_5 T_j) e^{-k_j z + i(qx-\omega t)} \\ &+ \sum_{j=1}^{3} B_j ((b_1 - b_2)iqu_j - b_4 k_j w_j - b_5 T_j) e^{k_j z + i(qx-\omega t)} \end{aligned} \tag{23}$$

$$\sigma_{xz}(x,z,t) = b_2 \left(\sum_{j=1}^{3} A_j (iqw_j - u_j k_j) e^{-k_j z + i(qx-\omega t)} - \sum_{j=1}^{3} B_j (iqw_j - u_j k_j) e^{k_j z + i(qx-\omega t)} \right) \tag{24}$$

where u_j, w_j, T_j, $j = 1, 2, 3$ are the corresponding eigenvector components of variables which satisfy $u_{j+3} = u_j$, $w_{j+3} = -w_j$ and $T_{j+3} = T_j$ for $j = 1, 2, 3$ from Eq. (16).

To complete the solution, the boundary conditions are that stresses and temperature gradient on the surfaces of the plate should vanish. We therefore have the following boundary conditions

$$\sigma_{zz} = \sigma_{xz} = \frac{\partial T}{\partial z} = 0 \quad \text{at } z = \pm d \tag{25}$$

Making use of the above boundary conditions, we obtain a system of six algebraic equations as follows

$$\sum_{j=1}^{3} ((b_1 - b_2)iqu_j - b_4 k_j w_j - b_5 T_j) \times (A_j e^{-k_j d} + B_j e^{k_j d}) = 0 \tag{26}$$

$$\sum_{j=1}^{3} ((b_1 - b_2)iqu_j - b_4 k_j w_j - b_5 T_j) \times (A_j e^{k_j d} + B_j e^{-k_j d}) = 0 \tag{27}$$

$$\sum_{j=1}^{3}(iqw_j - u_j k_j)(A_j e^{-k_j d} - B_j e^{k_j d}) = 0 \quad (28)$$

$$\sum_{j=1}^{3}(iqw_j - u_j k_j)(A_j e^{k_j d} - B_j e^{-k_j d}) = 0 \quad (29)$$

$$\sum_{j=1}^{3}(-k_j T_j)(A_j e^{-k_j d} - B_j e^{k_j d}) = 0 \quad (30)$$

$$\sum_{j=1}^{3}(-k_j T_j)(A_j e^{k_j d} - B_j e^{-k_j d}) = 0 \quad (31)$$

2.1. Frequency Equation

In order that the six boundary conditions be satisfied simultaneously, the determinant of the coefficients of the arbitrary constants must vanish. This gives an equation for the frequency of the plate oscillations. From Eqs. (26)–(31), we obtain the frequency equations in two factors for symmetric and antisymmetric modes of vibrations, respectively, as

$$\Delta_{\text{sym}} = E_1 F_1 \coth(k_1 d) - E_2 F_2 \coth(k_2 d) + E_3 F_3 \coth(k_3 d) = 0 \quad (32)$$

$$\Delta_{\text{anti}} = E_1 F_1 \tanh(k_1 d) - E_2 F_2 \tanh(k_2 d) + E_3 F_3 \tanh(k_3 d) = 0 \quad (33)$$

where $E_1 = (b_1 - b_2)iqu_1 - b_4 k_1 w_1 - b_5 T_1$, $E_2 = (b_1 - b_2)iqu_2 - b_4 k_2 w_2 - b_5 T_2$, $E_3 = (b_1 - b_2)iqu_3 - b_4 k_3 w_3 - b_5 T_3$, $F_1 = m_2 n_3 - m_3 n_2$, $F_2 = m_1 n_3 - m_3 n_1$, $F_3 = m_1 n_2 - m_2 n_1$, $m_1 = iqw_1 - u_1 k_1$, $m_2 = iqw_2 - u_2 k_2$, $m_3 = iqw_3 - u_3 k_3$, $n_1 = k_1 T_1$, $n_2 = k_2 T_2$ and $n_3 = k_3 T_3$.

The secular Eqs. (32) and (33) are the transcendental equations, which contain complete information about the phase velocity, wavenumber, and attenuation coefficient of the plate waves. In general, the wavenumber (and hence the phase velocity) of the waves is a complex quantity, and therefore the waves are attenuated. We can write $C^{-1} = V^{-1} + i\omega^{-1}Q$, so that $q = R + iQ$, where $R = \omega/V$, V and Q are real quantities, upon using above relation in the exponent of the plane wave solution given in Eq. (10) becomes $-Qx + iR(x - Vt)$. This shows that V is the propagation speed and Q is the attenuation coefficient of the waves. Using the value of C^{-1} given above, into frequency Eqs. (32) and (33), we can obtain the values of V and Q for different modes.

3. NUMERICAL RESULTS AND DISCUSSION

We consider Zinc material for the purpose of numerical calculations; the physical data for which is given as[64]

Table I. Coefficients of properties of Zinc material.

Symbol	Numerical value	Units	Symbol	Numerical value	Units
c_{11}	1.628×10^{11}	$(\text{N})(\text{m})^{-2}$	c_e	3.9×10^{2}	$(\text{J})^2(\text{kg})^{-1}(\text{deg})^{-1}$
c_{12}	0.362×10^{11}	$(\text{N})(\text{m})^{-2}$	ρ	7.14×10^{3}	$(\text{kg})(\text{m})^{-3}$
c_{13}	0.508×10^{11}	$(\text{N})(\text{m})^{-2}$	K_1	1.24×10^{2}	$(\text{kg})(\text{m})(\text{K})^{-1}(\text{s})^{-3}$
c_{33}	0.627×10^{11}	$(\text{N})(\text{m})^{-2}$	K_3	1.24×10^{2}	$(\text{kg})(\text{m})(\text{K})^{-1}(\text{s})^{-3}$
c_{44}	0.386×10^{11}	$(\text{N})(\text{m})^{-2}$	β_1	5.75×10^{6}	$(\text{N})(\text{m})^{-2}(\text{deg})^{-1}$
T_0	296	(K)	β_3	5.17×10^{6}	$(\text{N})(\text{m})^{-2}(\text{deg})^{-1}$
τ_0	0.05	–			

The calculation of the roots of the frequency equation for symmetric and antisymmetric modes represents a major task and requires a rather extensive effort of numerical computation. The complex secular equations for symmetric and antisymmetric modes have been solved by using the functional iteration numerical technique to obtain the values of non-dimensional phase velocity V and attenuation coefficient Q for different modes of wave propagation.

The non-dimensional phase velocity V and attenuation coefficient Q of wave propagation in the context of Lord-Schulman (LS) and the classical thermoelastic (CT) theories have been computed for various values of non-dimensional wave number R. The numerical results for symmetric and antisymmetric modes have been represented graphically in Figures 2–9. Figures 2 and 3 show the variation of non-dimensional phase velocity V with

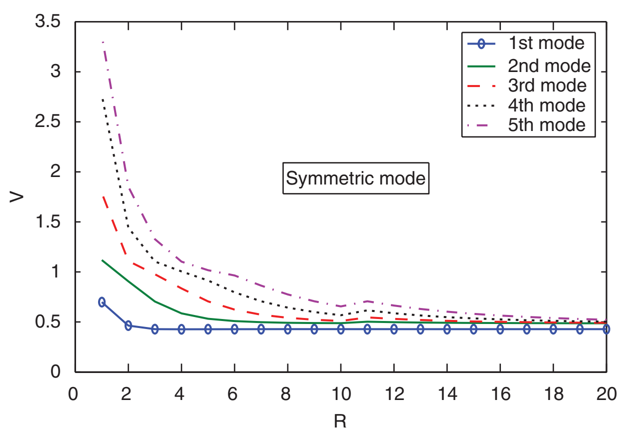

Fig. 2. Variation of phase velocity in case (LS) theory for symmetric modes of vibrations.

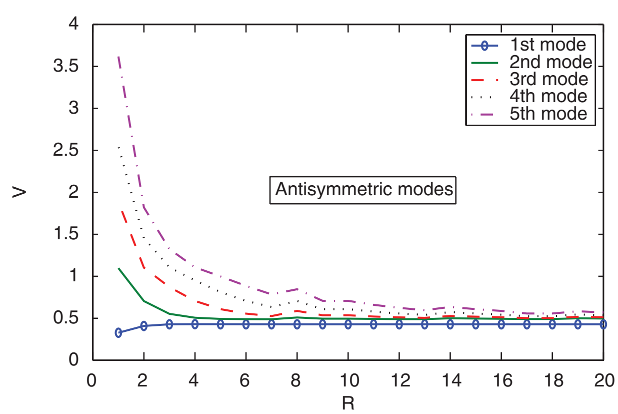

Fig. 3. Variation of phase velocity in case (LS) theory for antisymmetric modes of vibrations.

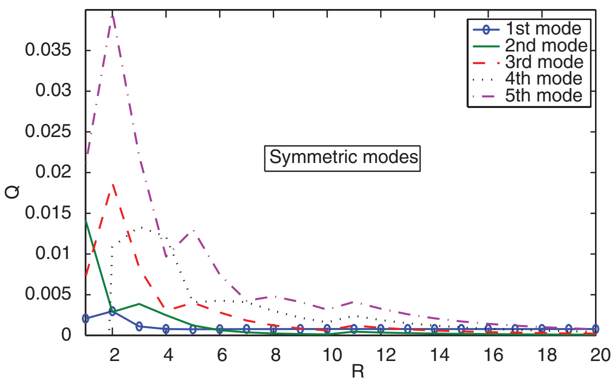

Fig. 4. Variation of attenuation coefficient in case (LS) theory for symmetric modes of vibrations.

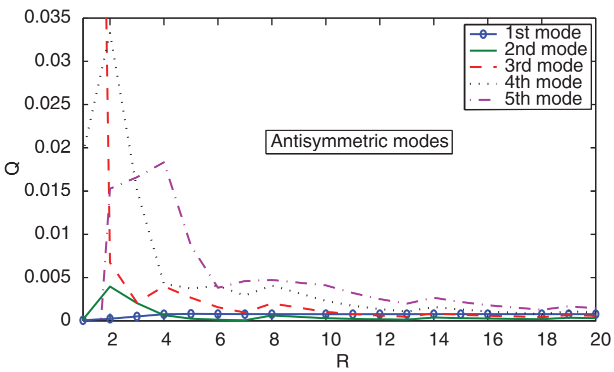

Fig. 5. Variation of attenuation coefficient in case (LS) theory for antisymmetric modes of vibrations.

respect to the wave number R, for the first five symmetric and antisymmetric modes respectively. As expected, the phase velocity decreases as the wave number increases, while the phase velocity increases with increasing the modes of wave propagation. Figures 4 and 5 show the variation of attenuation coefficients with respect to the wave number in the generalized thermoelastic plate for the first five symmetric and antisymmetric modes respectively.

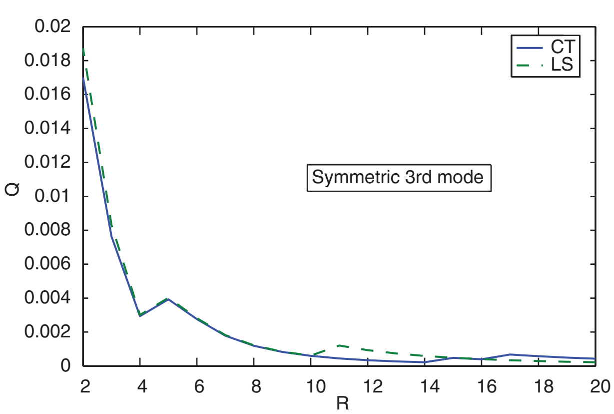

Fig. 6. Variation of attenuation coefficient in case symmetric third mode for different theories.

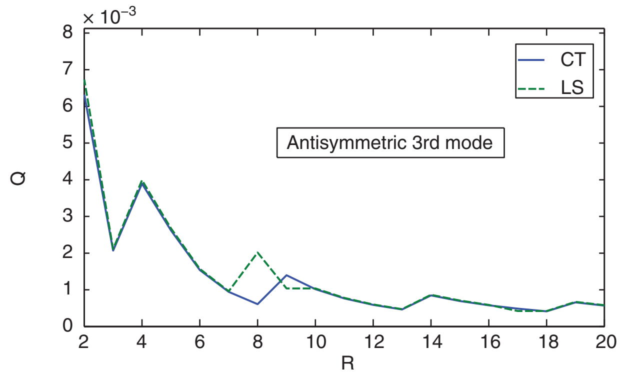

Fig. 7. Variation of attenuation coefficient in case antisymmetric third mode for different theories.

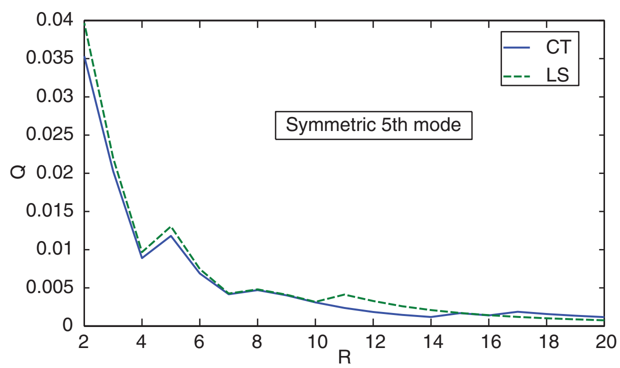

Fig. 8. Variation of attenuation coefficient in case symmetric fifth mode for different theories.

The magnitude of the attenuation coefficient increases monotonically, attaining the maximum values for the first five modes of wave propagation, and slashes down to become asymptotically linear in the remaining range of wave number. The generalized thermoelastic theory with one relaxation time (LS) is compared with the classical thermoelastic theory (CT) for symmetric and antisymmetric modes as in Figures 6 and 9. In these figures, the dashed lines refer to the generalized thermoelastic theory and the solid lines refer to the classical thermoelastic

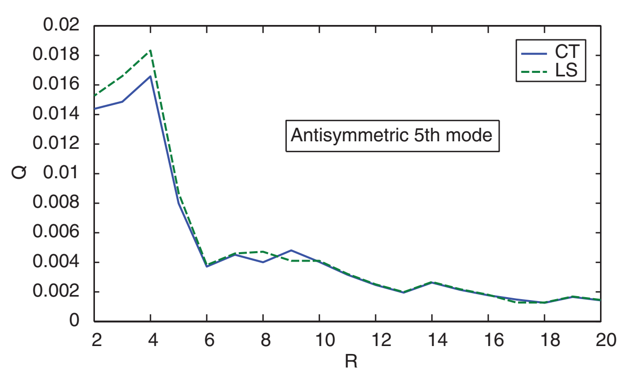

Fig. 9. Variation of attenuation coefficient in case antisymmetric fifth mode for different theories.

theory. In the case of the generalized thermoelastic theory, the values of attenuation coefficient are higher than that of the classical theory as in Figures 6–9.

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